

# A simplified method for predicting critical wind speed of cable-suspended bridge flutter at various angles of attack

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## SUMMARY:

In order to calculate flutter critical wind speeds (FCWSs) at various angles of attack (AoAs), it is found that the torsion-bending ratio is the main factor affecting the angle reduction coefficient. Based on this, a simplified method is proposed, which avoids complex iterative calculations and can calculate FCWSs directly. The acceptable results are obtained comparing predictions on FCWSs by the proposed method and the bimodal coupled bridge flutter theory for three cases from the existing cable-suspended bridge with difference structural characteristics.

Keywords: flutter, cable-suspended bridge, angle of attack

# **1. INTRODUCTION**

The cable-suspended bridge is a flexible system with cables as the main load-bearing structure and the deck laid directly on main cables, and the deck alignment is basically the same as the main cable alignment, as shown in Fig.1, which is used as a main infrastructure to connect towns and villages in mountainous areas of southwest China.



Figure 1. Cable-suspended bridge.

However, the complex wind environment in mountainous areas is often accompanied by large wind angles of attack (AoAs), making the wind resistance of cable-suspended bridges more prominent (Jing et al, 2020; Song et al, 2020), while it is difficult to conduct wind tunnel tests for every cable-suspended bridge due to the low engineering investment and the high cost of wind

tunnel tests. Hence, a simplified method for directly predicting the flutter critical wind speed (FCWS) of cable-suspended bridges at various AoAs is desperately needed. For research convenience, the aerodynamic shape of cable-suspended bridges is simplified to a flat plate. Based on this, the main factors of flutter critical wind speed at large AoAs are studied and a simplified method for evaluating flutter performance is proposed.

## 2. SIMPLIFICATION PROCEDURE

### 2.1. Angle reduction coefficient analysis

In Chinese specification (China, 2018), an angle reduction coefficient was introduced to consider the effect on FCWS caused by AoAs, which can be interpreted as the ratio of FCWSs with various AoAs to that at  $0^{\circ}$  AoA. The angle reduction coefficient can be expressed as:

$$R_{\alpha} = U_{cr}(\alpha)/U_{cr}(0^{\circ}) \tag{1}$$

$$U_{cr} = 0.416\omega_{\alpha}b_{\gamma}\left(1 - \frac{\omega_{h}^{2}}{\omega_{\alpha}^{2}}\right)\frac{mr}{\rho b^{3}}$$

$$\tag{2}$$

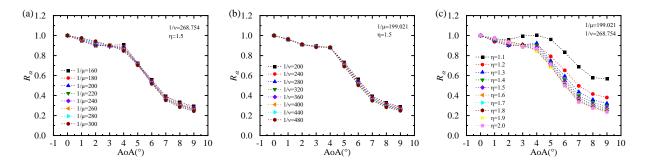
For flat plate, the FCWS at 0° AoA can be calculated from Selberg's formula (Chen, 2007), which is shown as Eq.(2). Where  $U_{cr}$  is the FCWS;  $\omega_{\alpha}$  and  $\omega_{h}$  are the circular frequency of first vertical and torsional modes; m is the mass per unit length;  $r = \sqrt{I/m}$  is the radius of gyration; I is the mass moment of inertia per unit length. Selberg's formula shows good applicability and accuracy in calculating the FCWS of flat plate at 0° AoA (Wu et al, 2020). From this perspective, the FCWS under various AoAs can be directly calculated by obtaining  $R_{\alpha}$ . In Chinese specification (China, 2018),  $R_{\alpha}$  was considered as a constant related to AoAs, while it was also determined by structural parameters for a defined aerodynamic shape of the bridge deck. Based on the bimodal coupled flutter analysis theory, the FCWS is mainly determined by dimensionless parameters  $\mu = \rho b^2/m$ ,  $\nu = \rho b^4/I$  and  $\eta = \omega_{\alpha}/\omega_h$ , if the structural damping is neglected and the aerodynamic shape is determined (Chen, 2007). Hence, the following analysis of  $R_{\alpha}$  will be carried out around these three dimensionless parameters. For cable-suspended bridges, because bridge decks are very similar, dimensionless parameters  $\mu$ and  $\nu$  remain essentially the same or vary within a small range. In contrast,  $\eta$  is mainly determined by the cable force and span and therefore varies considerably. The Niujiaoyan cablesuspended bridge in Guizhou, China, which had been damaged by strong winds on May 3, 2020, is used as the engineering background for the study. The structural parameters of the bridge are shown in case 1 of the Table 1.

Table 1	. Structural	parameters
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NO.	m(kg/m)	$I(\text{kg}\cdot\text{m}^2/\text{m})$	$f_h(\text{Hz})$	$f_{\alpha}(\text{Hz})$	<i>b</i> (m)	η	$1/\mu$	$1/\nu$
Case 1	1404.290	10922.850	0.360	0.429	2.400	1.193	199.021	268.754
Case 2	2480.420	3303.210	0.279	0.328	2.000	1.175	506.208	168.531
Case 3	2985.310	3975.590	0.266	0.346	2.000	1.300	609.247	202.836

For the reasons mentioned above, only smaller ranges of  $1/\mu$  and  $1/\nu$  were considered in the following analysis, while the frequency ratio  $\eta$  varies in a wide range and the variation of  $\eta$  is achieved by  $\omega_{\alpha} = \eta \omega_h$  and  $\omega_h$  remains unchanged. Additionally, closed-form solutions of the bimodal coupled bridge flutter (Chen and Kareem, 2006) are used to calculate FCWSs and the

results are shown in Fig.2. It should be noted that flutter derivatives of a flat plate under various AoAs were obtained by CFD methods.



**Figure 2.** Angle reduction coefficient: (a)  $\mu$ ; (b) v; (c)  $\eta$ .

As shown in Fig.2, the dimensionless parameters  $\mu$  and  $\nu$  had minimal effect on  $R_{\alpha}$  under the condition where the bridge aerodynamic shape remains unchanged, especially for the width of the bridge deck. The maximum difference of  $R_{\alpha}$  caused by the variation of  $\mu$  and  $\nu$  is within 5%, while that caused by  $\eta$  is over 30%. Hence, this provided a simplified method to calculate  $R_{\alpha}$  by considering  $\eta$  and neglecting the effects of  $\mu$  and  $\nu$ .

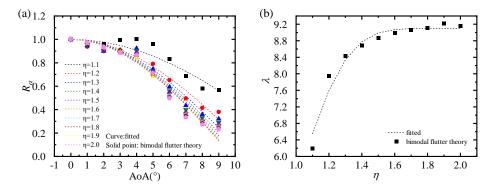
#### 2.2. Simplified formulations for flutter critical wind speed

According to Fig.2, it can be seen that  $R_{\alpha}$  is a function less than 1 and with respect to the AoA, and  $\eta$  seems to be a primary determinant of the function shape. From this point of view, a cosine function is used to fit, which can be expressed as:

$$R_{\alpha} = \cos\left(\lambda\alpha\right) \tag{3}$$

 $\lambda = 9.1 \left( 1 - 0.42^{\eta^2} \right) \tag{4}$ 

where  $\alpha$  is the AoA expressed in degrees;  $\lambda$  is the shape index corresponding to  $\eta$ , which can be obtained by Eq. (4). The fitting results of Eq. (3) and Eq. (4) are shown in Fig.3. It can be seen that the proposed formulas have more concise forms and can reflect the variation of  $R_{\alpha}$  to a certain extent. Therefore, the FCWSs at large AoAs can be easily estimated by combining Eqs. (2), (3) and (4) without the need for iterative calculations.



**Figure 3.** Fitting results: (a)Eq. (5); (b) Eq. (6).

## **3. NUMERICAL VALIDATIONS**

In order to validate the accuracy and effectiveness of the proposed formulas, three case studies are taken into account, which are reported in Table 1 originate from the actual cable-suspended bridges. Their FCWSs at various AoAs are calculated from both the proposed method and bimodal coupled bridge flutter theory, as shown in Fig.4. Although there are still differences between the results of the proposed method and the bimodal coupled bridge flutter theory, these differences are acceptable from the engineering application perspective. It is worth noting that the proposed method seems to overestimate FCWSs to some extent and requires highly similar bridge decks to guarantee the accuracy of the calculation.

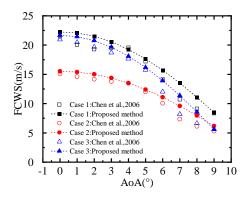


Figure 4. Results comparison.

# **4. CONCLUSIONS**

A simplified method is proposed to estimate the FCWS of cable-suspended bridges under various AoAs, which does not require iterative computation to obtain FCWSs. Although the results of the proposed method are somewhat different from the theoretical method and its use is limited, it provides a new way for calculating FCWSs at various AoAs especially for bridges with similar decks.

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#### REFERENCES

Chen, X.Z., Kareem, A., 2006. Revisiting multimode coupled bridge flutter: Some new insights. Journal of Engineering Mechanics 132, 1115-1123.

Chen, X.Z., 2007. Improved understanding of bimodal coupled bridge flutter based on closed-form solutions. Journal of Structural Engineering-ASCE 133, 22-31.

China, M.O.T., 2018. Wind-resistant Design Specification for Highway Bridges. Ministry of communications of the People's Republic of China.

Jing, H., Liao, H., Ma, C., Tao, Q., Jiang, J., 2020. Field measurement study of wind characteristics at different measuring positions in a mountainous valley. Experimental Thermal and Fluid Science 112, 109991.

Song, J.-L., Li, J.-W., Flay, R.G.J., 2020. Field measurements and wind tunnel investigation of wind characteristics at a bridge site in a Y-shaped valley. Journal of Wind Engineering and Industrial Aerodynamics 202, 104199.

Wu, B., Wang, Q., Liao, H., Li, Y., Li, M., 2020. Flutter derivatives of a flat plate section and analysis of flutter instability at various wind angles of attack. Journal of Wind Engineering and Industrial Aerodynamics 196.